# DOES FLEXIBILITY IN SECONDARY LEVEL MATHEMATICS CURRICULUM AFFECT DEGREE PERFORMANCE?

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## ABSTRACT

The authors have much experience in developing mathematics skills of first-year engineering students and attempting to ensure a smooth transition from secondary school to university. Concerns exist due to there being flexibility in the choice of modules needed to obtain a secondary level (A-level) mathematics qualification. This qualification is based on some core (pure maths) modules and a selection from mechanics and statistics modules. A survey of aerospace and mechanical engineering students in Queen's University Belfast revealed that a combination of both mechanics and statistics (the basic module in both) was by far the most popular choice and therefore only about one quarter of this cohort had studied mechanics beyond the basic module within school maths. Those students who studied the extra mechanics and who achieved top grades at school subsequently did better in two core, first-year engineering courses. However, students with a lower grade from school did not seem to gain any significant advantage in the first-year engineering courses despite having the extra mechanics background. This investigation ties in with ongoing and wider concerns with secondary level mathematics provision in the UK.

## KEYWORDS

mathematics, mechanics, curriculum, first-year, engineering

## INTRODUCTION

A decline, throughout the UK, in students' mathematical skills has been extensively described [1]. This problem is believed to have worsened since the early 1990s. More recently, however, another issue associated with students' mathematical ability on entering university has been highlighted. This relates to the structure of A-level Mathematics which is the qualification undertaken during the final two years at school.

There are various organisations in the UK responsible for the A-level curriculum, exams and assessment. One such body is the Council for the Curriculum, Examinations and Assessment (CCEA), based in Belfast, Northern Ireland. Its A-level Mathematics curriculum [2] involves four core modules of pure maths (C1 – C4) plus a selection of two out of four optional modules of applied maths (M1, M2, S1, S2).

Figure 1 summarises the content of these modules.

| C1 – Core Maths 1 | C2 – Core Maths 2 | C3 – Core Maths 3 | C4 – Core Maths 4 |
|---|---|---|---|
| Indices<br>Quadratic equations<br>Polynomials<br>Graphs of functions<br>Straight line<br>Differentiation | Circle geometry<br>Sequences<br>Series<br>Solving triangles<br>Trigonometry<br>Logs<br>Integration for area | Partial fractions<br>Parametric equations<br>Exponential function<br>Further calculus<br>Solving equations<br>Numerical integration | Functions<br>Differential equations<br>Volume of revolution<br>Further calculus<br>Vectors |

| M1 – Mechanics 1 | M2 – Mechanics 2 | S1 – Statistics 1 | S2 – Statistics 2 |
|---|---|---|---|
| Uniform acceleration<br>Force vector<br>Friction<br>Equilibrium – particles and rigid bodies<br>Newton's laws of motion<br>Impulse, momentum | Integration, differentiation of vectors<br>Projectiles<br>Circular motion<br>Potential, kinetic energy<br>Work-energy principle<br>Power | Presentation of data<br>Summary measures<br>Probability laws and functions<br>Discrete and continuous probability distributions<br>Normal distribution | Expectation algebra<br>Random sampling<br>Central limit theorem<br>Confidence intervals<br>Hypothesis testing<br>Bivariate distributions<br>Linear regression |

Figure 1. Summary of the syllabus for each of the A-level mathematics modules

The modules all have the same assessment weighting and therefore the A-level Mathematics qualification comprises two-thirds pure maths and one-third applied maths. The only permitted combinations of optional modules are: M1-M2, M1-S1, S1-S2. Within the mechanics and statistics streams of modules, the level of difficulty increases sequentially.

A separate A-level qualification in Further Mathematics involves an additional six modules, three of which are pure maths while the other three involve some combination of mechanics and statistics modules depending on which of these have already been taken for A-level Mathematics. Topics in further pure maths include matrices, determinants, groups, complex numbers, proof by induction and hyperbolic functions. Further mechanics topics include centre of mass, relative velocity, simple harmonic motion, satellite motion and Newton's law of restitution. There are no further statistics modules (in addition to those listed in Figure 1).

In the UK, most students enter university directly from school where they will have studied three A-level subjects in their final two years. For engineering students, these A-levels will probably include Mathematics and Physics. Our university selects students prior to enrolment on their degree programme by considering their A-level grades against published thresholds. Typically, a minimum of grade B will be required in Mathematics. However, given the various combinations of modules within A-level maths, described above, it is apparent that two students could have an A-level maths qualification, with the same grade, but have studied different topics. In an extreme example, a student could enter a mechanical engineering degree course having the required A-level maths qualification, but having not studied mechanics within A-level maths at school. A further issue is that the various exam boards offer slightly different syllabi.

This study aimed to investigate whether there was any difference in performance in first year for engineering students with different A-level maths module combinations.

Porkess [3] described the revised modular structure of A-level mathematics just before its introduction in September 2004. He regretted that there was no longer an equal balance between pure and applied maths and noted that, overall, there had been a reduction in content. However, he believed the new arrangements presented an opportunity for students to take Further Mathematics, at least to AS-level, in addition to Mathematics and thereby broaden their mathematical knowledge. Note that AS is the first half of an A-level and can be taken as a separate qualification. Thus, while studying for three A-levels is the norm, many students take the opportunity to do an AS-level in a fourth subject.

Robinson et al [4] considered the decline in applied mathematics content at A-level with particular focus on mechanics. A survey of 242 schools across England showed a relatively high proportion (26%) of schools offered only the basic mechanics module (M1), at most, to students studying A-level maths. A survey of first-year engineering students at three English universities showed a wide range of mechanics backgrounds with about 10% of students having not studied any mechanics modules within A-level maths. The authors expressed concern that many university academics were unaware of the mechanics knowledge of incoming students and were not monitoring developments in A-level mathematics.

Lee et al [5] reported that a mechanics diagnostic test successfully discriminated between first-year engineering students in terms of the number of mechanics modules they had studied at school. The authors recommended such testing at entry to university and offering appropriate follow-up support.

A comprehensive and more recent report [6] on the "mechanics problem" detailed a significant decline in students studying mechanics at A-level since the curriculum changed in 2004. For example, numbers of students taking mechanics module M2 dropped by about 30% in subsequent years. In 2006, only 22% of A-level maths students (using the Edexcel exam board) took M2. With a requirement to select only two applied modules, the most popular choice by schools was M1-S1 (the basic mechanics and statistics modules) in order to maximise students' grades and expose students to a wider range of applications.

The above data may give an overly pessimistic view with regard to students proceeding to study engineering at university. The report quotes a study at the University of Bristol where 58% of engineering students in 2006 had studied M2, down from 72% in the previous year [7]. Studying AS-level Further Mathematics alongside A-level Mathematics was recommended [6] as a realistic means to accessing more mechanics modules and thereby being better prepared to undertake an engineering degree.

The above concerns are likely to be shared internationally. A major study across four Australian universities investigating factors responsible for high fail rates in first-year engineering mechanics courses [8] concluded that the problem was complex, with a wide range of topics identified as problematic for students, but that students' school grades correlated only weakly with introductory mechanics results at university. Another Australian study [9] demonstrated that students with higher level maths from school did significantly better in first-year mechanics at university than those without but that physics background was not as good a predictor of performance in mechanics at university. However, various key topics in mechanics were challenging for all students, independent of academic background.

In the literature from the USA, there is a focus on developing concept inventory assessment tools to test students' understanding or ability to apply key concepts and to identify for teachers those topics requiring greater weight [10, 11].

## METHODOLOGY

The study has involved a survey of aerospace and mechanical engineering students who began their degrees at Queen's University Belfast in September 2011 and who have therefore completed first year. There were 52 aerospace students and 95 mechanical students although they have been treated here as a single population given that the two degree programmes have identical entry requirements and a broadly similar first-year curriculum. A-level maths module choices were surveyed by contacting the students directly – it seems that only the A-level grade is provided to the university.

Students' results in the first-year Solids & Structures and Engineering Dynamics modules were analysed. Solids & Structures is a first-semester course representing one-twelfth of the first-year curriculum. It covers plane frames, shear force and bending moment, torsion, stress-strain relations and beam bending stress, slope and deflection. Engineering Dynamics is a second-semester course, also weighted as one-twelfth of first year. It covers Newton's laws of motion, moment of inertia, linear and rotary systems, variable mass problems and an introduction to mechanical vibrations. These modules provide a foundation for more advanced study in structures and dynamics in subsequent years. It was hypothesised that any gaps in students' knowledge of mechanics, through focusing more on statistics at A-level, would have a particular effect in these important modules – hence, they were chosen for analysis.

## STUDENT BACKGROUND IN MATHEMATICS

Figure 2 shows the mathematics qualifications of the incoming students. It is noted that a small proportion does not have A-level maths – these students will have studied alternative courses, perhaps at a further education college, containing some elements of maths and deemed by the university to be a suitable alternative route on to the degree programme. It is also notable that only a small group has taken the more advanced Further Mathematics in addition to their Mathematics A-level. Also, the AS-level Further Mathematics has not proved popular – it was hoped [3] that many students would take this course in addition to A-level Mathematics in order to supplement their mathematics background.

Discussions between the author and local school teachers suggest that mathematically-able pupils are more likely to proceed and complete A-level Further Mathematics rather than stop at the AS-level stage. Other pupils with an interest in science/engineering may consider that a broader range of subjects offers greater flexibility with regard to future career options.

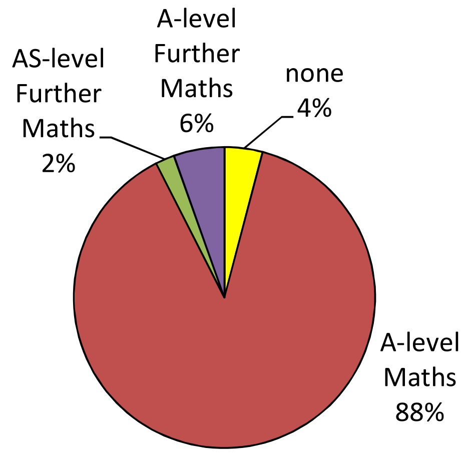



Figure 2. Students' highest A-level mathematics qualification

Figure 3 shows the grades attained by students in A-level Mathematics. As expected, these are mainly A*, A and B grades due to the minimum requirements for admission to a university engineering degree course.

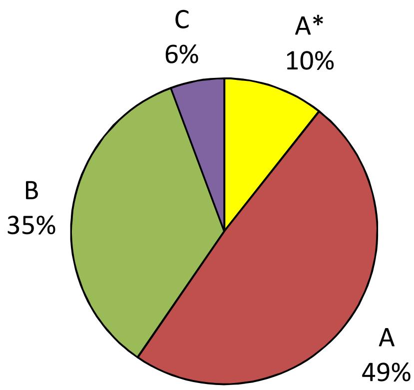



Figure 3. Students' grades in A-level Mathematics

Figure 4 shows the exam boards which provided the A-level Mathematics courses. A large majority of students in these degree courses are from Northern Ireland and many local schools use the CCEA exam board (indicated by NI in Figure 4). About one third of students did the course provided by Edexcel, an exam board based in England.

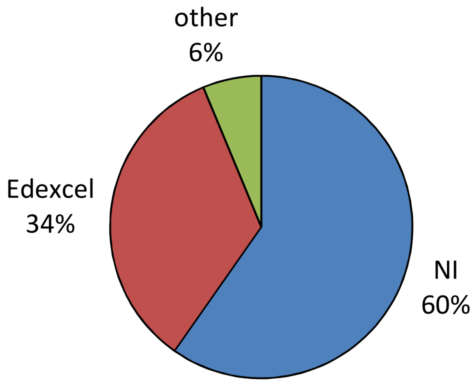
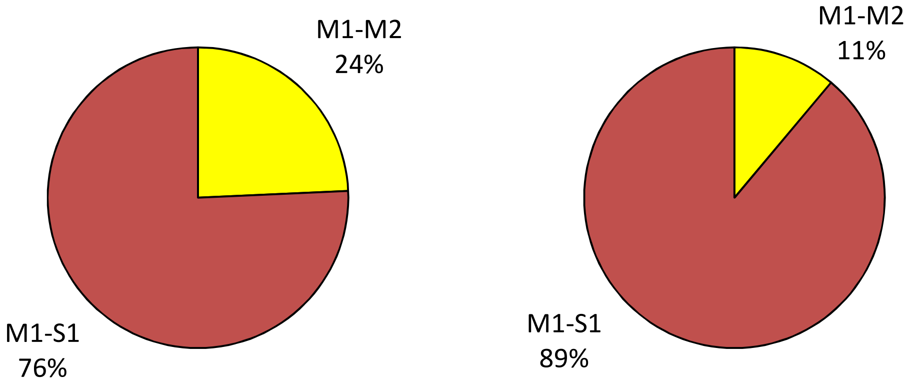



Figure 4. Exam boards from which students' took A-level Mathematics

The A-level Mathematics module combinations taken by the students are shown in Figure 5 with the data for the two main exam boards presented separately. The eleven Further Mathematics students, who would have taken a much wider range of modules, are not included in this data. Another small group has also been omitted, including those whose module combination is unknown. However, the data give a good overview of the maths background of the majority of the class (the A-level Mathematics only students). The popularity of the M1-S1 combination is striking, as is the absence of any student taking the statistics-only combination (S1-S2). Overall, the data indicate that about one quarter of this particular class studied mechanics beyond the basic module within school mathematics.

In discussions with the author, local school teachers asserted that the M1-S1 combination is easier than M1-M2. Thus, M1-S1 is an attractive option for schools which desire high grades to benefit both their pupils and the school's reputation. A teacher from a school which usually selects M1-M2 believed this combination to be more sensible – the emphasis on mechanics fits neatly alongside A-level Physics which is also taken by most of the A-level Mathematics pupils.




(a) NI exam board (b) Edexcel exam board

Figure 5. Module combinations for students taking A-level Mathematics from the NI and Edexcel exam boards

## ANALYSIS OF FIRST-YEAR PERFORMANCE

Figures 6 and 7 present students' results in the first-year Solids & Structures and Engineering Dynamics modules, comparing them with A-level Mathematics grades. While there is a general trend, it is clear that A-level Mathematics grade is not a good predictor of performance in these modules. For example, even those students who achieved one of the top grades in school (grade A) scored across a very wide range (30% – 90%) in both Solids & Structures and Engineering Dynamics.

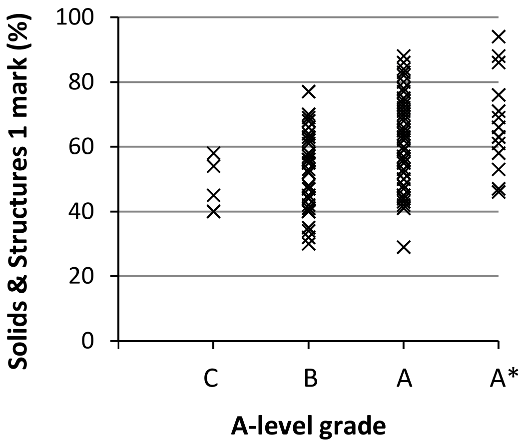



Figure 6. Comparison of students' results in first-year Solids & Structures with their A-level Mathematics grades

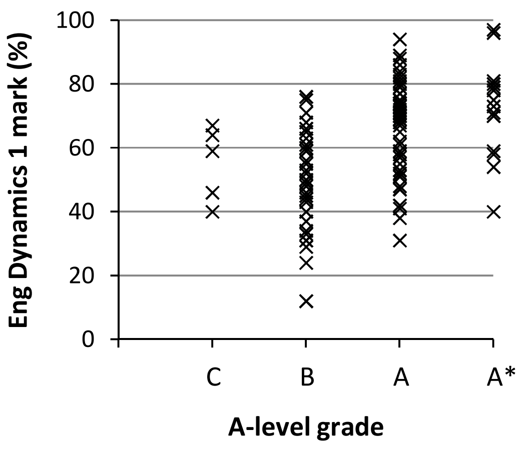



Figure 7. Comparison of students' results in first-year Engineering Dynamics with their A-level Mathematics grades

Tables 1 and 2 separate the students by A-level Mathematics module combination in an attempt to identify whether this has any effect on first-year performance. The data is restricted to students whose maths was provided by the NI exam board to eliminate any effects due to differences in syllabi. Students with A-level Further Mathematics were omitted, as were those who had transferred to aerospace/mechanical engineering having previously attempted university first year elsewhere, in order to eliminate anyone with extra exposure to mechanics compared to those taking the traditional route from A-level Mathematics straight into the first year of the aerospace/mechanical degree. This left a group of 70 students, 60 of whom had also studied A-level Physics. The sample was therefore reduced to those 60 students to diminish the effects of different backgrounds in physics. Of these, 15 students had done M1-M2 and 45 students had done M1-S1.

In all but one case, for students with a particular A-level Mathematics grade, those who did the M1-M2 combination did better (on average) that those with the M1-S1 background. Indeed, the grade A M1-M2 students did better (on average) than the grade A* M1-S1 students in both modules. The differences in the averages were significant only for the grade A students (in Solids & Structures) and the grade A* students (in Engineering Dynamics), and the degree of significance was small. It is noted, however, that sample sizes are small. For the grade B students, those with a greater mechanics background did not do significantly better than those without. This suggests that only the students who are stronger academically were able to make good use of their extra background in mechanics and gain an advantage in these first-year modules. It also raises questions about the depth of understanding in mechanics achieved by the grade B students who took M1-M2. It is reiterated that sample sizes were small – data collection from the current class has already begun to allow further investigation.

Table 1. Comparison of average marks in Solids & Structures with students grouped by A-level Mathematics grade and A-level Mathematics module combination

| A-level Maths grade | average mark in Solids & Structures 1<br>A-level Maths module combination: | | significant difference? |
|---|---|---|---|
| | M1-M2 | M1-S1 | |
| A* | 74.5 | 62.6 | no |
| A | 64.8 | 57.3 | yes, $p < 0.1$ |
| B | 51.0 | 51.6 | no |

Table 2. Comparison of average marks in Engineering Dynamics with students grouped by A-level Mathematics grade and A-level Mathematics module combination

| A-level Maths grade | average mark in Engineering Dynamics 1<br>A-level Maths module combination: | | significant difference? |
|---|---|---|---|
| | M1-M2 | M1-S1 | |
| A* | 77.0 | 66.4 | yes, $p < 0.05$ |
| A | 70.8 | 66.7 | no |
| B | 60.0 | 48.3 | no |

**CONCLUSIONS**

This paper has reported the different mathematics backgrounds of students taking aerospace and mechanical engineering degrees at our university. Only a small proportion (6%) had achieved A-level Further Mathematics and studying for the AS-level Further Mathematics was rare (2% of this class). A combination of mechanics and statistics was the most popular choice of optional modules in A-level Mathematics, meaning that only about one quarter of the class had studied mechanics beyond the basic module within school mathematics.

A-level Mathematics grade was not a good predictor of performance in the first-year Solids & Structures and Engineering Dynamics courses. However, when students were separated according to A-level Mathematics module choice, benefits were observed for those who had focused on mechanics (M1-M2) but, interestingly, these benefits seemed to be restricted to the academically-stronger students (grades A and A*).

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## BIOGRAPHICAL INFORMATION

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