

# ACTIVE LEARNING IN REDESIGNING MATHEMATICS COURSES FOR ENGINEERING STUDENTS

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## ABSTRACT

“Prepare, Participate, Practice”: active learning in designing basic maths courses for engineering students at TU Delft works! The PProject Innovation Mathematics Education (PRIME) at Delft University of Technology (TU Delft) is all about redesigning mathematics courses for engineers. This paper describes the process of developing, implementing, evaluating and implementing again of three basic courses at TU Delft using a blended learning approach developed by a growing team of teachers from the mathematics department. Our findings suggest that the approach taken enhances students’ learning performance in maths education. The main results show that students have a more active learning experience compared to the traditional setup of these courses, leading to more engagement, more interaction and better results. An important role is played by meaningful examples taken from the engineering faculty where the students are studying, showing students from that faculty what role the mathematics play in their field of interest. This is also used to develop their skills in mathematical modelling.

## KEYWORDS

Engineering education, blended learning, mathematics, team-based development, active learning, CDIO- Standards: 1, 2, 8, 9, 10, 11, 12.

## INTRODUCTION

In this paper we consider interfaculty education: mathematics for non-mathematics students at TU Delft. Students need to have a sound mathematical background to pursue their studies and in their future careers. Pinxten (2018) shows that students need 6 to 8 hours of mathematics training in secondary education each week and a sufficient to very good grade at the final exam to have a chance of success in studying Engineering. Continuation of

diligent study time in mathematics is a necessity for any engineering student to obtain their bachelor degree and achieve academic success.

Mathematics at TU Delft is taught within the engineering faculties before or parallel to the disciplinary courses in the engineering programmes. It allows the students, or so it is hypothesized, that students use the mathematical theory and apply it in their disciplinary engineering assignments. Despite the high expectations, the transfer of theory to practical application in the disciplinary field is limited, as shown by student evaluations, passing rates and lecturer reports. From studies in childhood mathematics learning it is known the more concrete object and materials are used to learn mathematics the more difficult the transfer becomes of the mathematics to other disciplinary or isomorphic assignments. Abstract mathematics allows for better transfer and better ability to understand relational structures, allowing for math skills transfer to alternative math topics. (Kaminsky & Sloutsky, 2012). Concrete objects increases the salience of superficial aspect and divert the attention from the relational structures to be learned. The more complex the problems become the more susceptible to diverted attention the learner is.

Finally, student engagement and intrinsic motivation are stimulated by establishing more autonomous learning, a feeling of competency (self efficacy) and relatedness to other students who may struggle with the same materials (Deci & Ryan (2002) Bandura, (1997),Artino (2012)). The present situations allow for little to no autonomy as the programme is fixed and a schedule to be met. Once the students are behind there is little time or possibility to catch up, bearing on the feelings of competencies. Frequent testing overburdens the students and possibly makes them loose their intrinsic motivation and engagement with the mathematics material.

To solve the issues mentioned above, a new teaching approach was developed in “PRIME” . In this paper the following questions are researched upon: Does the new teaching method activate/engage students (more), does it improve transfer, does it improve passing rates? First we start by describing the project. Next the development of the new approach and the didactical concept chosen are reflected upon. Then the implementation of the concept is reported, followed by the consequences and improvements implemented after the first operation of the courses. Data analysis of the results over the past two years are presented and finally some suggestions for future research are discussed.

## **THE PROJECT: PRIME**

In 2014, PRIME (PROject INNovation MAThematics EDUcation) was initiated in order to conceive a different approach to the math courses for engineering students.

### ***The organization of PRIME***

The initial project team consisted of a group of six dedicated lecturers from Delft Institute of Applied Mathematics (DIAM), an e-learning developer, an educational advisor and a project leader. The project was supported by the Executive Board of the university. A large steering group was assigned to the project to keep informed about the progress: it consisted of the Vice-President of Education (Executive Board), the director of Student Affairs, the dean of the faculty of Applied Physics, the dean of the faculty of Electrical Engineering, Mathematics and Computer Science (EEMCS), the director of education of the faculty of Aerospace

Engineering, the director of education of the faculty of EEMCS, the chair of the Mathematics department, a student from the Mathematics student association. After two years of running the project, the team has expanded into a team of a senior project leader, an assistant project leader, 12 instructors, an e-learning developer, an educational advisor and four student assistants. The steering group has remained the same, except for the student member, who has been replaced by two students: one from Civil Engineering and one from Aerospace Engineering. The steering committee gathers once every three to four months with the project management team.

### ***The goals of PRIME***

Three goals were formulated:

1. *Academic success*: to improve study results
2. *Transfer*: to improve the connection between mathematics and engineering
3. *Engagement*: to increase students active participation in class and motivation for the topic

In the following subsections each of the measures taken to address these goals is described briefly.

#### *Academic success*

Once the student is motivated for mathematics, the next important challenge is to activate him: active learning enhances retention and improves understanding of subsequent subjects in the student's learning path (Veenstra- van Dijk, 2000). Moreover, it is well known that mathematics needs practice, in order to acquire the skill of interacting in a mathematical way with their disciplinary field of study, needed to learn new concepts.(Kirschner et al., 2006)

#### *Transfer*

Showing the use of mathematics in the field of interest of the student is believed to enhance motivation for learning (Chickering et al., 1987). With the help of lecturers and students from the receiving faculties, contextual examples from the specific fields are worked out, to illustrate the use of mathematics in the field of interest. Finding examples that are interesting, not too hard to explain for the mathematicians, not too hard to understand for the students turned out to be a challenge. A new smaller project carried out by Cabo & Makaveev (2018) has resulted in a new method to investigate the use of mathematics in specific engineering courses. The lessons learned from this project will be implemented in PRIME shortly. They involve also incorporating projects in a later stage of the courses to apply their knowledge in practice, an important feature of engineering education (Edström, 2008; Kamp, 2016).

#### *Engagement*

*Engagement can be defined as the extend to which students actively participate in learning activities (online presences, watching video's and doing assignments) and face to face contact meetings (coming to class, being prepared, make use of the materials to digest the learning materials). It equally includes the stimulation of student motivation by relating abstract materials to their disciplinary field of study The extent to which students are engaging in higher education is supposed to strengthen the learning outcomes. (Trowler, V, 2010, HEA report)*

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### ***The courses innovated in PRIME***

To start with three basic maths courses were considered for innovation: Calculus 1 and 2, Linear Algebra (both first year courses) and Probability & Statistics (first or second year course). Since the context examples are tailored to each individual program, the courses are not exact copies of each other. However the content is mostly exchangeable, only the pace of each course may differ. Bachelor programs with courses in PRIME are Aerospace Engineering, Computer Science, Electrical Engineering, Civil Engineering and Mechanical Engineering. In the near future courses like Differential Equations and Calculus 3 in certain programs will also be innovated. A typical course consists of nine weeks of two or three two-hour lectures, resulting in 18 to 27 contact hours.

### **BLENDED LEARNING CYCLE: “PREPARE, PARTICIPATE, PRACTISE”**

A number of educational principles have been included to achieve the innovation and goals of the project. Active participation in teaching sessions (Freeman et al, 2014), conceptual understanding in the face to face contact (Rittle- Johnson et al., 2015), adequate performance feedback (Hattie 2007, Boud & Falchnikov, 2006) and a carefully balanced format of contextual examples (Cabo & Makaveev, forthcoming) using contextual problems, with a sufficient level of generalisation, to motivate the importance of maths in other fields of study and equally support transfer. In other words: the student should **prepare** himself before coming to class, actively join in-class- activities and after the face-to-face session – **participate** and **practice** to process the new knowledge. Due to the workload of teachers and increasing student numbers, the stimulation of autonomous learning, competency building and time on task, a blended approach was felt to best meet the requirements (Bonk et al., 2006; Szeto, 2014). A video has been recorded, available on TU Delft website (2017), to stimulate students to study differently.

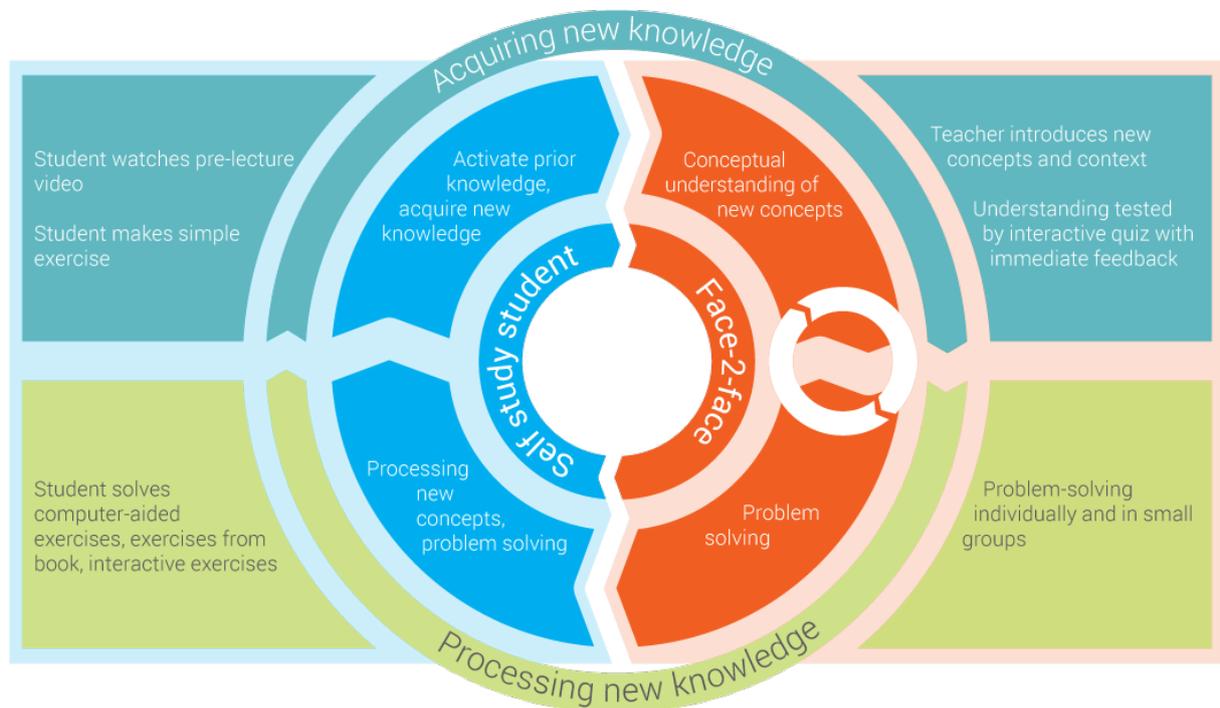


Figure 1: Blended learning cycle used in PRIME

### **Practise**

At home (or wherever the students wants), a set of computer aided exercises can be done: an online platform offers two or three types of exercises: basic, intermediate (with an optional help function to guide the student through the exercises) and an assignment, to be handed in online. At the moment a platform offered by an editor is being used, however the project management is currently looking for an (open) alternative.

### **BLENDED LEARNING: MATERIAL DEVELOPED (hybrid flipped classroom)**

This model of blended learning is established as a sort of hybrid flipped classroom, as shown in the sequence prepare, participate and practice, the flipped model of autonomous learning and reflection and discussion in class to further explore the learning materials is not enough. The practice step consolidation of the learned materials is essential to bring the math skills to the next level of learning.

For each course new material has been developed by the project team. During frequent meetings (once every one or two weeks), first a lesson plan was designed, with all the learning outcomes listed. Then consensus had to be reached on which learning outcome could go into the pre lecture video, and how the others would be covered in the slides. Exercises had to be chosen, contextual examples had to be collected from the faculties and implemented into the course. An overview of the course, linking the separate subjects was constructed, and included in the collaborative learning environment, showing students how the subjects connect.

Evaluation and evolution of all the learning material is constantly being done: lecturers send their comments to a special mailbox created for this. If possible changes are implemented immediately by the student assistants. More drastic improvements are collected and stored. During meetings where new courses are being prepared for their pilots, every remark on the content, video, course structure, exercises and quizzes is taken into account, discussed, reviewed and altered if necessary.

### Using the Collaborative learning environment

All the material for the course is presented to the students in a well-organized page on Brightspace, the collaborative learning environment in use at TU Delft since September 2017. The lectures are structured by week and represent the blended learning cycle.

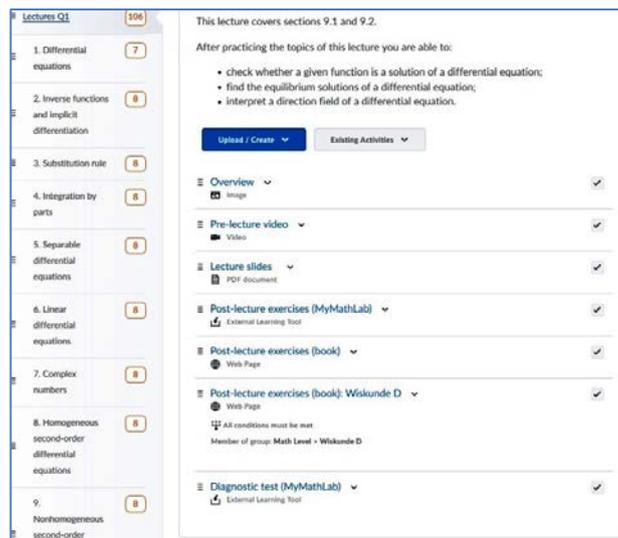


Figure 2. Example of a lesson on Brightspace

### Overview

A graph representing an overview of the subjects presented in the course, shows students the connection between different subjects covered

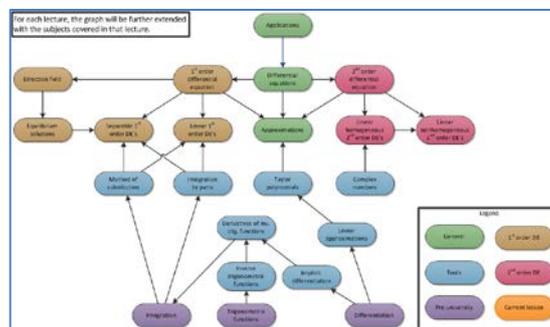


Figure 3. Overview of a course Calculus 1 (Civil Engineering)

### **Sub-parts of the Course design**

The course consists of online exercises to practice the conceptual understanding of the subject together with the book exercises. The exercises provide feedback and allow repetition as much as needed by the students. 110 videos have been recorded covering an introductory subjects, half of them are used as a type of homologation in which students secondary education knowledge is upskilled (TU Delft, 2018). A slide pack is the framework/benchmark for all the lecturers and students involved. It includes definitions, theorems, contextual examples, interactive quiz questions, workflow of a lecture, accomplished learning goals after having done all the lectures activities and homework. Finally, there are interactive quiz questions including questions on conceptual understanding. Depending on the results of the quiz, the lecturer can decide to further elaborate on the subject and stimulates active participation of students in class. It is reported by lecturers and students that the quizzes stimulate active participation of students in class.

### **Mathematical modelling**

One of the learning goals of the newly designed courses in Calculus was to teach the students the mathematical modelling cycle: this is the most important application of their mathematical knowledge in practice.

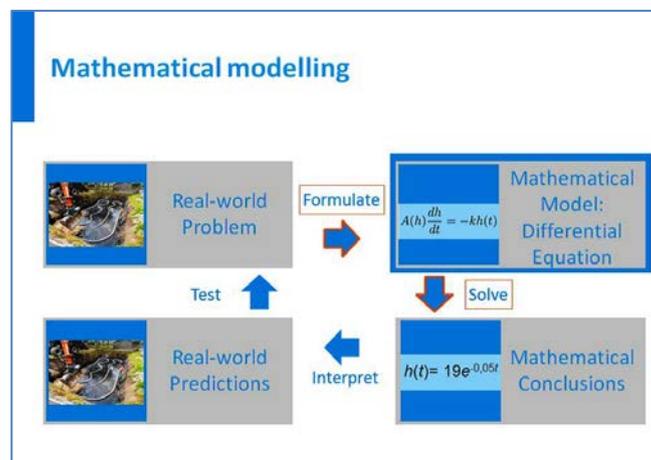


Figure 4. Mathematical model cycle

### **EVALUATION**

The evaluation was focused on whether the new teaching method activated/engage students (more), is transfer improved and are the passing rates improving? The data are as much as possible triangulated and emerge from data at the programme level, the lecturers and the student evaluation. At this point we were not yet able to formulate any research hypothesis.

### **Program directors and academic success**

The program directors of the Bachelor curricula involved are pleased with the innovaton: the activity of the students has increased, and – after an initial dip in the results- the study success rate has increased (Table 1) They appreciate the fact that mathematical modelling is

now part of the learning outcomes, and they hear from lecturers of their own faculty that they feel more comfortable about the expected level of mathematical background of the students of their own classes.

Table 1. Passing rates

	2013/14	2014/15	2015/16	2016/17
<b>LinearAlg AE</b>	61%	72%	52%	75%
<b>Prob&amp;Stat AE</b>	54%	19%	56%	67%
<b>Prob&amp;Stat EE</b>	67%	79%	54%	70%
<b>Calculus 1 CE</b>	73%	68%	64%	68%

### ***Lecturers role in the hybrid flipped model***

The impact on lecturers involved in blended learning has been investigated in different ways. After the first pilot a survey was distributed among the nine lecturers who taught the course. (Vos, 2016). In subsequent courses, for each course three meetings were held to discuss the content and impact of the course: one before the course started, one in the middle of the course (week 4 or 5 of the quarter) and one at the end, after the course had finished, but before the exam was taken. The instructors commented on the use of the pre-lecture videos and how to deal with the fact that students don't watch them: 50 % of the instructors tend to repeat the material of the videos, 50 % does not, or in a concealed way. The teachers are positive about the interactive quizzes although some of them (40%) thinks it takes too much of their instruction time. Using the slide pack is for 40 % of the teachers a burden: they are used to teach the course in their own way. The other 60% however think it is helpful to reduce preparation time. All teachers have seen that the students are more active during the classes, and the attendance is higher than it was before the blended teaching. Working in a team to develop and discuss course content was appreciated by 70% of the lecturers. The support from the project lead was considered sufficient (70%), could have been more (30%). The cultural change needed in the teaching staff turns out to be a tough process. It takes more time to get the teachers along than it takes to convince the students. Observing each other's classes was viewed as a relevant and stimulating experience, helping to improve the quality of teaching.

### ***Students Engagement***

Apart from the official quality cycle (Evasys) - a survey that students fill out after having done the exam (average response rate 30%)- the project management implemented the so-called Stop-Start-Continue Survey. In this questionnaire the students are asked to write down what they would like the lecturer to start doing, stop doing or continue doing. The survey is given to the students during class, the response rate is quite high (70 - 90%). The general remarks collected from this survey are grouped and the ones that appear the most are commented on by the responsible lecturer together with the project management. These comments are

posted on Brightspace. The comments that relate to individual teachers are sent to the teachers, and they discuss them in class.

In the second quarter of the academic year 2017-2018 a lunch meeting with students from Civil Engineering, with part of the project team and the responsible lecturer was organized to discuss the outcomes of the survey. The use of contextual examples was highly appreciated there. The students confirmed that they liked the way of teaching and the videos, but also gave some useful feedback on individual teachers and explanation of the online exercises.

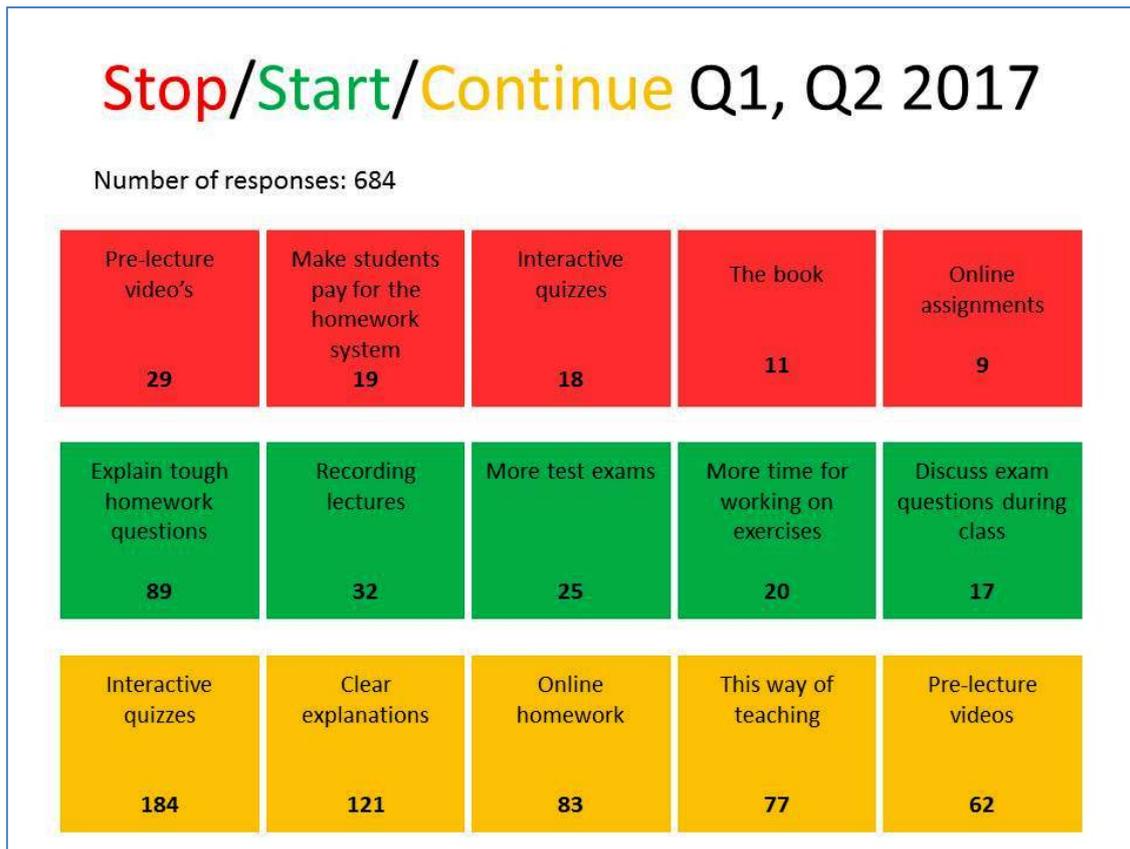


Figure 5. Outcome of the StopStartContinue Survey in the 1<sup>st</sup> semester of 2017-2018

### **Transfer**

Most important reviews were on the contextual examples: some of them were too difficult to understand for the students, some of them were too difficult to explain for the teachers, some were not realistic enough. Also some videos had to be recorded anew because they had too much content. Furthermore, a lot of the interactive questions were adapted because either they did not connect well enough to the videos, or they did not test concepts well enough or they took too much time to answer.

### **CONCLUSIONS**

After three runs of the courses Calculus 1 for Civil Engineering and Probability and Statistics for Electrical Engineering, they seem to have reached a steady state. The rest of the courses, that have run two times, or only one time, need adjustments.

Working in large teams of teachers improves the quality of the courses and the consensus on how to teach the course, this is noticed by the students. Blended learning is welcomed by students, blended teaching is a challenge for some of the teachers. Finding suitable and meaningful examples to illustrate the use of mathematics is a tough challenge. Help from students from the relevant programs might turn out to be crucial to improve this. In one instance (Cabo, 2018) this turned out to be the solution. On the other hand the use of this kind of examples in the courses is really appreciated by the students.

## **FUTURE DEVELOPMENTS AND RESEARCH**

In the near future, the courses are being improved using student evaluations and teacher experiences. It is interesting to find out if three runs with this intensity of adjustment and evaluation is the standard to get to a steady state situation of a course. Additionally, many research questions have emerged as a result from designing and re-designing these courses.

A lot of data is being collected from the students. Well-defined research questions should guide the relevance of the learning analytics data gathered until now and from the next academic year onward. In particular we will investigate how individual learning paths enhance student's learning, and how active learning effects the understanding of the mathematics taught.

In the academic year 2017-2018 a pilot has been done at Civil Engineering by grouping students having a similar, somewhat better, mathematical background from secondary school: Did these students perform better in the mathematics course in higher education than their less prepared counterparts? Did they appreciate the extra information and deepening of the learning experience they were offered? Is it worthwhile expanding this experiment to other faculties?

Teachers that are late adapters or have problems getting used to this PRIME approach will be supported with extra training activities. This will contribute to lifelong learning and faculty development on teaching mathematics in the PRIME model.

The lessons learned from the project investigating how to better implement the connection between mathematics and aerospace engineering, will be incorporated in PRIME and further expanded. What the most efficient way is to embed discipline based examples in mathematics or computational learning is to be explored.

The ambition is to involve the multiple stakeholders in the data collection and analysis to generate more evidence based support for the things that have intuitively been done right and extend this to a larger community within TU Delft and beyond.

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**Renate Klaassen**, Dr. is an educational consultant, working at the TU Delft Teaching and Learning Services. She has been heavily involved in educational advising on the innovation of the BSc in Aerospace Engineering, and various other curriculum reforms at TU Delft. Currently, she is TU Delft Programme Co-ordinator and Researcher of the 4TU.Centre for Engineering Education. Consultancy activities include assessment (policy, quality and professionalization), internationalisation of university education and design education. Areas of research interest pertain to content, language integrated learning in higher education, Conceptual Understanding in Engineering Education and Interdisciplinary learning.

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